Design and Optimization of Multi-ring Permanent Magnet Bearings for High-speed Rotors- A Computational Framework

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Abstract

This article presents a computational framework (MATLAB app) suitable for the industrial use for selecting optimum multi-ring radial and thrust permanent magnet bearings (PMB) based on two general variables (outer diameter/air gap and length of a bearing). Such an approach eliminates the usage of complex design equations and optimization methods. The detailed methodology adopted in optimizing PMB for maximum characteristics is presented with mathematical equations of force and stiffness. Then, the steps involved in the development of the computational framework are discussed in depth. Further, usage of the computational framework is explained with examples of PMB, and results obtained are validated with the mathematical model results. Regarding the mathematical model results, deviations of 2.22 % and 1.78 % are observed among the maximized radial and axial force values in the app results. Finally, the effectiveness of the proposed framework is demonstrated by discussing the case studies from the literature.

Keywords: MATLAB app; Design; Optimization; Axial air gap; Radial and thrust bearings.
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1. Introduction

Magnetic bearings are friction-free bearings wherein attractive or repulsive magnetic forces levitate the moving part. The lubrication-free operation, long life, and less maintenance are the exciting features offered by these bearings. Magnetic bearings are of two types; active and passive. In active magnetic bearings (AMB), the rotor is levitated using electrical and electronic components with an external energy source whereas, levitation of the rotor is achieved by arranging permanent magnets without any external energy sources in passive magnetic bearings.[12] Passive magnetic bearings could be realized by using permanent magnets or permanent magnets and conductor (eddy current effect) or superconductors.[13-14] Applications like energy storage flywheels, conveyor, and turbo compressors, permanent magnet bearings have been used effectively to replace conventional bearings like ball and roller bearings.[7-10] Earlier, researchers contributed to developing two-dimensional (2D) equations to calculate force and stiffness by assuming the axisymmetric of the bearing model for a rotating shaft.[11,12] Delamare et al. synthesized and classified the different possible elementary configurations of PMB.[13] Force and stiffness characteristics were calculated using the dipole method. The authors developed 2D analytical equations for radial stiffness and load in stacked-structure PMB.[14] Later, the authors reported three-dimensional equations in the literature by accounting for the curvature effect of rings in complicated equations consisting of the first and second kinds of elliptical integrals.[15-16] A simple vector approach for calculating PMB features was also presented using Coulombian model.[17-20] To address the issue associated with the lower values of force and stiffness characteristics of PMB with only one ring on the rotor and stator, stacked structures (multiple rings on the rotor and stator) were introduced.[21] This necessitates generalized equations for bearing features in stacked structures with ‘n’ number of rings (multi-ring PMB).[22-24] Optimization of geometrical parameters was performed to improve the characteristics of multi-ring PMB as comparable with conventional bearings. K. P. Lijesh and H. Hirani presented optimization of axially polarized monolithic radial PMB for maximum radial force in a particular magnet volume using modified 2D equations, and the design procedure for the same monolithic bearing was presented.[25,26] Optimization of radial PMB was demonstrated for maximum
load-carrying capacity using 2D finite element analysis (FEA) and presented the generalized curves for the optimized parameters.[27] The authors optimized multi-ring PMB using 2D analytical equations for maximum bearing characteristics.[28,29] Conventional and rotation magnetized direction configurations of multi-ring thrust PMB were optimized by Bekinal et al. using 3D equations by taking the effect of the curvature of the rings into account.[30,31] Lijesh et al. presented the complete optimization of stack structured radial PMB for maximum load-carrying capacity using a single objective function and for both optimum radial force and stiffness with the help of multi-objective function.[32,33] Permanent Magnet Bearing configurations were optimized by providing an air gap between successive rings for optimum bearing features using 2D equations, and results were validated with 3D FEA.[34,35] An optimization methodology for multi-ring radial and thrust PMB was presented in our earlier efforts.[36] Following are the observations based on the review of literature on design, optimization, and selection of multi-ring PMB,

- Researchers used multi-ring PMB to replace conventional bearings used in different applications.
- Multi-ring PMB was designed and optimized to have characteristics comparable with the conventional bearings.
- Design and optimization of PMB involve solving complicated non-linear equations of force and stiffness using mathematical tools (e.g., MATLAB or Mathematica).

Above mentioned steps imposed difficulty for the designer in selecting optimum PMB in different applications. These difficulties could be addressed by developing a MATLAB app to support the selection process in the industry concerning the general requirements. This paper presents the MATLAB app for the selection of optimum PMB in different applications. Mathematical equations for evaluating force and stiffness generated in multi-ring radial and thrust PMB are presented in section 3. Then the design, optimization, generalization of an optimization procedure for PMB in a magnet volume is in section 4. Section 5 highlights the procedure adopted in developing the MATLAB app, and application examples and case studies are discussed in section 6.

2. Multi-ring radial and thrust bearing configurations
Neodymium iron boron (NdFeB) permanent magnet rings can be arranged adjacent to the other on the rotor and stator surfaces to form multi-ring structures with the bearing characteristics comparable with the conventional bearings. Axially polarized rings are arranged in alternate opposite polarizations to realize radial bearing as shown in Fig. 1(a). By reversing the polarization directions of either rotor or stator rings of radial bearing configuration, the thrust bearing can be formed as shown in Fig. 1(b).

3. Mathematical approaches
Magnetic interactions between faces of rings in one of the pairs in multi-ring structures are shown in Fig. 2. The interactions forces between the faces of magnets are calculated using Coulombian model and vector approach. The position vectors between the respective faces are shown in Fig. 2. The total force in the multi-ring structure is due to the interaction of each face of rotor rings with all faces of stator rings. Generalized semi-analytical equations of forces and stiffness in multi-ring radial and thrust bearings are presented in Eqs. (1) to (5).

The radial force, Fr is expressed as:

\[
F_r = \frac{B^2}{4\pi \mu_0} \sum_{d=1}^{n} \sum_{c=1}^{m} \sum_{b=1}^{n_m} S_{c(1-f)} \sum_{d=1}^{m} \sum_{b=1}^{n_b} \frac{d c}{R_{r}^2} R_{r}(e(a)(f b)) (-1)^{(c+d)} (-1)^{(a+b)}
\]  

(1)

![Fig. 1](image-url) Multi-ring permanent magnet bearings (a) radial bearing (b) thrust bearing.
where, $R_{(e)(f)(db)} = \sqrt{(X_{db} - X_{eca})^2 + (Y_{db} - Y_{eca})^2 + (Z_{db} - Z_{eca})^2}$ and $R_{(eca)(f)(db)} = (X_{db} - X_{eca})i + (Y_{db} - Y_{eca})j + (Z_{db} - Z_{eca})k$.

The procedure adopted in designing and optimizing permanent magnet radial/thrust bearing is presented curve fit equations in step 8. A new permanent magnet radial bearing configuration with different (g/D4) values. The relationships between optimized parameters and air gap values. A different set of optimized radial/thrust bearing parameters are shown in Fig. S1.

3. Optimum force and stiffness values are calculated in the selected volume of a magnet by varying the number of rotor and stator rings using the MATLAB codes developed for solving Eqs. (1) to (5). Then the optimum number of rings ‘no’ is decided based on the maximized values of force and stiffness. (Detailed explanation of this step for a particular set of parameters is represented in the form flow chart as shown in Fig. S1).

4. By fixing the ‘no’, an air gap between the adjacent rings in the axial direction is varied and corresponding force and stiffness values are calculated using modified MATLAB codes of Eqs. (1) to (5). Optimum axial air gap value ‘ago’ is decided based on the values of force and stiffness.

5. By fixing ‘no’ and ‘ago’, the inner diameter of rotor rings (D1) is varied, and corresponding force and stiffness values are calculated. The optimum value ‘D1o’ is decided based on the values of force and stiffness.

6. Then, the inner diameter of stator rings (D3) is varied by keeping ‘no’, ‘ago’ and ‘D1o’ constant, and corresponding values of force and stiffness are calculated. Based on the results obtained, the optimum value ‘D3o’ is decided. The value of ‘D2o’ is also determined from ‘g’ and ‘D3o’. A detailed explanation of optimizing D3 for a particular set of parameters is represented in the form flow chart as shown in Fig. S2.

7. Steps (2) to (6) are repeated for different radial air gap ‘g’ values. A different set of optimized values are available at this stage corresponding to the number of ‘g’ values selected.

8. The relationships between optimized parameters and air gap are plotted concerning the outer diameter of the stator rings, and curve fit equations for the same are presented for different (g/D4) values.

9. A new permanent magnet radial bearing configuration with different aspect ratios (L/D4) varying from 0.25 to 1.5 in steps of 0.25 and different air gap values from 0.25 to 2.5 in steps of 0.5 mm is selected.

10. Values of all optimized design parameters are determined for all selected aspect ratios and air gaps using the presented curve fit equations in step 8.
11. In a selected bearing configuration, maximized force and stiffness (Frm or Fzm & Krm or Kzm) values are calculated by considering all optimized parameters (obtained in step 10) using Eqs. (1) to (5) for all aspect ratios and air gap values. Force and stiffness (Frms or Fzms & Krms or Kzms) values are also calculated by assuming monolithic bearing configuration in the selected volume of the magnet.

12. The relationship between (Frm or Frms)/(Fzm or Fzms) & (L/D4), and (Krm or Krms)/(Kzm or Kzms) & (L/D4) are plotted, and curve fit equations for the same are expressed for different ratios of (g/D4).

13. The curve fit equations presented in step 12 can be used to calculate the final maximized force and stiffness values of the optimized configuration by using the ratios (g/D4) and (L/D4) and by calculating force (Frms or Fzms) and stiffness (Krms or Kzms) values of monolithic bearing configuration.

5. MATLAB computational framework

App designer is a toolbox developed by MATLAB to create a graphical user interface (GUI). An App designer consists of a self-contained MATLAB program that automates a task or calculation. With the help of an App designer, a user can create professional applications in MATLAB. In addition, various features available in the App designer can be employed to efficiently work the computational framework. These multiple features, also known as visual components, can be dragged from the component library and dropped into the user interface figure (Design view) to be displayed to the user. An integrated editor (code view) can be used to program the user behaviour of every component. The advantage of using App designer is that for most of the components available in the component library, the codes are predefined, which reduces the time and effort in coding every component used in creating an app. This application can be used as a web app or even a desktop app, provided the user's system has an inbuilt MATLAB compiler. In creating the present application in App designer, three windows are considered: procedure, optimization of the thrust bearing, and optimization of the radial bearing. First, in the procedure window (Fig. 3), the instructions for managing the application framework, the range of L/D4 value to be considered for the calculations, and the option for selecting the bearing type are provided on the screen.

The second window (refer Fig. 4) is for optimizing the thrust bearing, and it provides many tabs such as the input, g/D4, calculate force or stiffness, optimized results for the user. In addition, a figure of the thrust bearing configuration is provided for the user’s reference. The input tab contains four edit fields; g, D4, L, and Br.

A ‘g edit field’ and 'D4 edit field' accept the user's input and store it for future calculations. The callback function is assigned to the 'L edit field,' which indicates an error message if the condition of L/D4 (within 0.2 to 1.5) does not satisfy. The callback functions assigned to the 'Br edit field' also show an error message if the user does not enter Br value. The user is restricted to enter only the 'g' value or 'D4' value. If the user enters both the values, then the callback function assigned to the g and L edit fields will throw an error message instructing the user to enter any one value only. The 'g/D4 tab' will be enabled once the required input is entered in the input tab, and it consists of five buttons that show the value of g/D4 available for the calculations. Also, every button on this tab is assigned with a callback function which turns the button colour to green on clicking it. Thus, if the user enters the 'D4' value and clicks on any one of the five buttons in this tab, the callback function

![Design and Optimization of Permanent Magnet Bearings](image)

**Fig. 3** The procedure window of an app.

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will calculate and give the output of the 'g' value in the 'g edit field'. If the user enters the 'g' value and clicks on any of the five buttons in this tab, the callback function will calculate and give the 'D4' value output in the 'D4 edit field'.

Two buttons are provided in the calculate tab, namely 'force' and 'stiffness.' Once the user clicks on these buttons, MATLAB will do the corresponding calculations in the back end. For example, the callback function assigned to the force button consists of the set of equations and mathematical models that calculates the optimized parameters of the bearing, such as \( H_0, N_0, D_{1o}, D_{2o}, D_{3o}, D_{4o}, F_{zm}/F_{zms}, F_{rm}/F_{rms}, F_{ms}, F_{zms}, F_{zm}, \) and \( F_{rm}. \) Similarly, the callback function assigned to the stiffness button consists of the set of curve fit equations and mathematical models that calculates the optimized parameters of the bearing, such as \( K_{zm}/K_{zms}, K_{rm}/K_{rms}, K_{ms}, K_{zms}, K_{zm}, \) and \( K_{rm}. \) A message box appears on the screen, asking the user to wait until the back-end calculations are complete. Once the calculations are done in the back end, a dialogue box pops up on the screen indicating the user to click on the 'optimized stiffness button' or 'optimized force button' depending on the input given in the previous step (i.e., if the user has selected the 'force button' then he is supposed to click 'optimized force button' and vice versa). After clicking the 'optimized force' or 'optimized stiffness button,' all the optimized results will be displayed on the optimized results tab at the screen's bottom left. A 'reset button' is also provided to clear all the current screen values and carry out the next set of computations. A 'procedure button' is also given if the user wants to switch to the main menu (i.e., procedure window).

In the third window (refer Fig. 5), all the components related to optimization of the radial bearing are set up similar
to the second window (i.e., optimization of the thrust bearing). Here in this window, the image is replaced with a radial bearing configuration. Calculations will also change as per the curve fit equations and mathematical models assigned in the code view (.mapp), an extension for the App designer file. All the files associated with creating this application framework in the App designer are to be set in the same MATLAB path; otherwise, it will show an error.

6. Results and discussions
In this section, the developed MATLAB app for designing and optimizing multi-ring radial and thrust PMB is demonstrated with examples of bearing with different values aspect ratios (L/D4) and remanence of the magnets (Br).

Example 1
Variables D4 =0.2 m, L= 0.15 m, g/D4=0.01 and Br =1.2 T are chosen for designing and optimizing radial PMB for maximum radial force. The obtained values of optimized parameters in the selected magnet volume and maximized radial force in single and multi-ring radial bearings by running the app are shown in Fig. 6. The maximized values of forces are also calculated by solving Eq. (1) in MATLAB, and results are plotted in Fig. S4. Results obtained using the app match closely with mathematical model results, and a deviation of 2.22% among maximized radial forces is observed. The radial force of an optimized configuration is 24 times the force generated in monolithic bearing.

Example 2
For designing radial PMB for maximum stiffness, the variables D4 =0.1 m, L= 0.1 m, g/D4=0.02 and Br =1.2 T are selected. The obtained values of optimized parameters in the selected magnet volume and maximized radial stiffness in single and multi-ring radial bearings by running the app are shown in Fig. S3. The maximized stiffness values are also calculated by solving Eq. (5) in MATLAB, and results are plotted in Fig. S4. Results obtained using the app match closely with mathematical model results, and a deviation of 0.87% among maximum stiffness values is observed. The radial stiffness of an optimized configuration is 21 times the stiffness generated in monolithic bearing.

Example 3
For designing thrust PMB for maximum axial force, the variables D4 =0.08 m, L= 0.08 m, g/D4=0.025 and Br =1.4 T are selected. The obtained values of optimized parameters in the selected magnet volume and maximized axial force in single and multi-ring thrust bearings by running the application are shown in Fig. S5. The maximized values of forces are also calculated by solving Eq. (3) in MATLAB, and results are plotted in Fig. S6. Results obtained using the app match closely with mathematical model results, and a deviation of 1.32% among maximum force values is observed. The axial force of an optimized configuration is 8.6 times the force generated in monolithic bearing.

Example 4
Variables D4 =0.08 m, L= 0.06 m, g/D4=0.0125 and Br =1.4 T are chosen for designing and optimizing thrust PMB for maximum axial stiffness. The obtained values of optimized parameters in the selected magnet volume and maximized axial stiffness in single and multi-ring thrust bearings by running the app are shown in Fig. S7. The maximized stiffness values are also calculated by solving Eq. (5) in MATLAB, and the results are plotted in Fig. S8. Results obtained using the app match closely with mathematical model results, and a deviation of 1.78% among maximum stiffness values is observed. The axial stiffness of an optimized configuration is 16.9 times the stiffness generated in monolithic bearing. Based on the optimized results obtained using the app and mathematical model for all possible cases of

![Fig. 6 Optimized results of radial PMB for maximum radial force using app.](image-url)
designing radial and thrust PMB with different Br values, the app could be used effectively in different applications without the effort of solving complex design equations and optimization procedures.

The effectiveness of using the application framework for the design and optimization of PMB is demonstrated by considering the analysis of configurations published in the literature.

**Case study 1**

Detoni *et al.* [37] used multi-ring radial PMB to support the rotor with the dimensional details presented in Table 1. By selecting D4=0.03385 m, L=0.0177 m, and Br=1.18 T, radial PMB is designed and optimized for maximum stiffness using the application, and results are given in Table 1 and Fig. 8. The radial stiffness of an optimized one is 1.92 times the configuration proposed in [37], and the number of optimized rings is 8 instead of 6.

**Case study 2**

In this, thrust PMB presented in [22] is selected for the application's design and optimization. The authors calculated the axial force for the dimensions given in Table 2 and by selecting D4=0.052 m, L=0.02 m, and Br=1.13 T, the

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**Fig. 7** Results of the maximized radial force using the mathematical equations.

**Fig. 8** Design and optimization of radial PMB presented in [37] using the app.

**Table 1.** Optimized details of radial PMB presented in [37].

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<thead>
<tr>
<th>Radial bearing details of [37]</th>
<th>Optimized results using the proposed computational framework</th>
</tr>
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<tbody>
<tr>
<td>D1=0.0173 m, D2=0.0253 m, D3=0.0273 m, D4=0.03385 m, n=6, L=0.0177 m and Br=1.18 T. Radial stiffness, Kr=91000 N/m</td>
<td>D1o=0.0104 m, D2o=0.0270 m, D3o=0.0287 m, D4o=0.03385 m, n=8, ago=0.0004 m, L=0.0177 m and Br=1.18 T, Max. radial stiffness, Kr=174300 N/m</td>
</tr>
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Optimization is carried out using the app, and results are given in Table 2 and Fig. 59. An enhancement of 22.7% is observed in the maximum axial force value in the optimized configuration.

### 7. Conclusions

Based on two general parameters, a MATLAB desktop app for selecting optimum multi-ring radial and thrust PMB is presented. The usage of the application framework for selecting optimum radial PMB for maximum radial force and stiffness is demonstrated, and results are validated with the mathematical model results. The maximized radial force and stiffness of the optimized configurations are 24 and 21 times the values of the monolithic bearing. Using the proposed application, the selection of optimum thrust PMB for maximum axial force and stiffness is demonstrated with an example, and it is observed that the axial force and stiffness of the optimized configurations are 8.6 and 16.9 times the values of the monolithic configuration. Based on the optimized results obtained from the proposed computational framework in case studies, radial stiffness is enhanced by 91.5%, and thrust force by 22.7% in the same magnet volumes of the selected configurations. The proposed computational framework could calculate optimum dimensions and maximum characteristics of radial and thrust PMB in one step just by selecting two general parameters representing aspect ratio (L/D4) in the industry.

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### Conflict of interest

There are no conflicts to declare.

### Supporting information

Applicable.

### References


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